# Analyse

## Exam

28th of June of 2007

1 Let $X$ be a compact subset of $\mathbb{R}^n$. Prove

(i) $X$ is closed and bounded,

(ii) if $A$ is a closed subset of $X$, $A$ is compact.

(2 points)

2 Consider $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = \frac{yx^2+y^4}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

(i) Is $f$ continuous at $(0,0)$? (give an appropriate argument.)

(ii) Is $f$ differentiable at $(0,0)$? (give an appropriate argument.)

(2 points)

3 Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = x^3 y$

(i) write down the the definitions of integrability and differentiability,

(ii) prove that $f$ is differentiable at $(a,b) \in \mathbb{R}^2$,

(iii) use the definition of integrability to compute the integral of $f$ in $X = [0,1] \times [0,1]$. (Hint: prove first that $1^3 + 2^3 + 3^3 + \ldots + n^3 = (1+2+3+\ldots+n)^2 = (\frac{n(n+1)}{2})^2$)

(4 points)

4 Write down the implicit function theorem. (A proof of it is not required.) (1 point)